

2 sem 1562

Exo 1 :

① $f(x, y) = \frac{1}{4}x^2 y$, $g(y) = 6x + 3y = 60$

$\mathcal{L}(x, y, \lambda) = \frac{1}{4}x^2 y - \lambda(6x + 3y - 60)$

$\frac{\partial \mathcal{L}}{\partial x} = \frac{xy}{2} - 6\lambda = 0 \Rightarrow \frac{xy}{2} = 6\lambda$

$\frac{\partial \mathcal{L}}{\partial y} = \frac{x^2}{4} - 3\lambda = 0 \Rightarrow \frac{x^2}{4} = 3\lambda$

$\begin{cases} \frac{xy}{2} = 6\lambda \\ \frac{x^2}{4} = 3\lambda \end{cases} \Rightarrow \frac{xy}{2} = \frac{x^2}{2}$

$\Rightarrow \boxed{y = x}$

on a $6x + 3y = 60$

$\Rightarrow 6x + 3x = 60 \Rightarrow \boxed{x = \frac{20}{3}}$

$\boxed{y = \frac{20}{3}}$ (0,5)

(0,5)

② $f(x, y) = -3x - 4y + 6$

$g(x, y) = x^2 + y^2 = 1$

$\mathcal{L}(x, y, \lambda) = -3x - 4y + 6 - \lambda(x^2 + y^2 - 1)$

$\begin{cases} \frac{\partial \mathcal{L}}{\partial x} = -3 - 2\lambda x = 0 \\ \frac{\partial \mathcal{L}}{\partial y} = -4 - 2\lambda y = 0 \end{cases} \Rightarrow \begin{cases} 2\lambda = -\frac{3}{x} \\ 2\lambda = -\frac{4}{y} \end{cases}$

$x^2 + y^2 - 1 = 0$

$\Rightarrow \frac{3}{x} = \frac{4}{y} \Rightarrow \boxed{y = \frac{4}{3}x}$

$x^2 + y^2 - 1 = 0 \Rightarrow x^2 + \left(\frac{4}{3}x\right)^2 = 1$

$\Rightarrow \frac{25}{9}x^2 = 1 \Rightarrow x_{1,2} = \begin{cases} +\frac{3}{5} \\ -\frac{3}{5} \end{cases}$

$x_1 = \frac{3}{5}$

① $\lambda = \frac{-3}{2x}$

② $y = \frac{4x}{3}$

• pour $x = \frac{3}{5}$

$\lambda = \frac{-3}{2 \cdot \frac{3}{5}} \Rightarrow \boxed{-\frac{5}{2} = \lambda}$

• pour $x = -\frac{3}{5}$

$\lambda = \frac{-3}{2 \cdot (-\frac{3}{5})} \Rightarrow \boxed{\lambda = \frac{5}{2}}$

• $x = \frac{3}{5}$ et $y = \frac{4x}{3}$

$\Rightarrow y = \frac{4}{3} \cdot \frac{3}{5} \Rightarrow \boxed{y = \frac{4}{5}}$

• $x = -\frac{3}{5}$

$\Rightarrow y = \frac{4}{3} \cdot \left(-\frac{3}{5}\right) \Rightarrow \boxed{y = -\frac{4}{5}}$

on a deux points critiques :

$\left(\frac{3}{5}, \frac{4}{5}, -\frac{5}{2}\right)$ et $\left(-\frac{3}{5}, -\frac{4}{5}, \frac{5}{2}\right)$
 $\underbrace{\left(\frac{3}{5}, \frac{4}{5}, -\frac{5}{2}\right)}_{(x_1, y_1, \lambda_1)} \quad \underbrace{\left(-\frac{3}{5}, -\frac{4}{5}, \frac{5}{2}\right)}_{(x_2, y_2, \lambda_2)}$

①

$$(3) f(x, y) = x + 2y$$

$$g(x) = x^2 + y^2 - 1$$

$$L = x + 2y - \lambda(x^2 + y^2 - 1)$$

$$\frac{\partial L}{\partial x} = 1 - 2\lambda x = 0 \rightarrow x = \frac{1}{2\lambda}$$

$$\frac{\partial L}{\partial y} = 2 - 2\lambda y = 0 \rightarrow y = \frac{1}{\lambda}$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - 1 = 0 \quad (1)$$

$$ds(1) : x^2 + y^2 - 1 = 0$$

$$\Rightarrow \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 = 1$$

$$\Rightarrow \frac{1}{4\lambda^2} + \frac{1}{\lambda^2} = 1$$

$$\frac{5}{4\lambda^2} = 1 \Rightarrow \lambda = \pm \frac{\sqrt{5}}{2}$$

• pour $\lambda = \frac{\sqrt{5}}{2}$:

$$\text{on a } x = \frac{1}{2\lambda} = \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)}$$

$$\Rightarrow \boxed{x_1 = \frac{\sqrt{5}}{5}} \quad \boxed{y_1 = \frac{2\sqrt{5}}{5}}$$

• pour $\lambda = -\frac{\sqrt{5}}{2}$

$$\boxed{x_2 = -\frac{\sqrt{5}}{5}} \quad \boxed{y_2 = -\frac{2\sqrt{5}}{5}}$$

deux points critique

$$\left(\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{2}\right) \text{ et } \left(-\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{2}\right)$$

$$(4) f(x, y, z) = (x-3)^2 + (y-4)^2 + (z-12)^2$$

$$g(x, y, z) = x^2 + y^2 + z^2 - 4$$

$$L = (x-3)^2 + (y-4)^2 + (z-12)^2 - \lambda(x^2 + y^2 + z^2 - 4)$$

$$\frac{\partial L}{\partial x} = 2(x-3) - 2\lambda x = 0 \rightarrow x = \frac{3}{1-\lambda}$$

$$\frac{\partial L}{\partial y} = 2(y-4) - 2\lambda y = 0 \rightarrow y = \frac{4}{1-\lambda}$$

$$\frac{\partial L}{\partial z} = 2(z-12) - 2\lambda z = 0 \rightarrow z = \frac{12}{1-\lambda}$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 + z^2 - 4 = 0 \quad (1)$$

$$ds(1) = \left(\frac{3}{1-\lambda}\right)^2 + \left(\frac{4}{1-\lambda}\right)^2 + \left(\frac{12}{1-\lambda}\right)^2 = 4$$

$$169 = [2(1-\lambda)]^2$$

$$\Rightarrow \lambda_1 = \frac{15}{2} \quad \lambda_2 = -\frac{11}{2}$$

• pour $\lambda = \frac{15}{2}$ $\left(x = \frac{3}{1-\lambda}, y = \frac{4}{1-\lambda}, z = \frac{12}{1-\lambda}\right)$

$$\Rightarrow \boxed{x_1 = -\frac{6}{13}} \quad \boxed{y_1 = -\frac{8}{13}} \quad \boxed{z_1 = -\frac{24}{13}}$$

• pour $\lambda = -\frac{11}{2}$

$$\boxed{x_2 = \frac{6}{13}} \quad \boxed{y_2 = \frac{8}{13}} \quad \boxed{z_2 = \frac{24}{13}}$$

Exo 2:

Le temps $t = \frac{\text{distance}}{\text{vitesse}} = \frac{e}{v}$

$$t_1 = \frac{e_1}{v_1} = \frac{\sqrt{x^2 + 1}}{3}$$

$$t_2 = \frac{e_2}{v_2} = \frac{2-x}{6} \quad (2)$$

$$t_{\text{total}} = t_1 + t_2 = \frac{\sqrt{x^2 + 1}}{3} + \frac{2-x}{6}$$

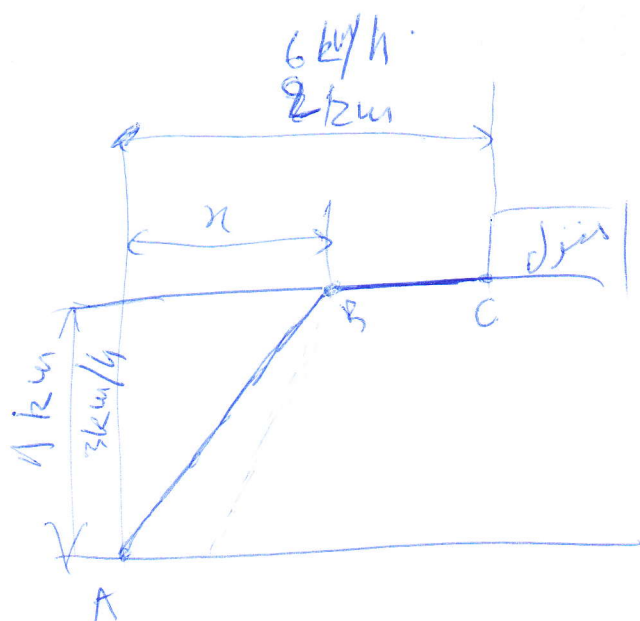
$$(2) \quad t'(x) = \frac{x}{3\sqrt{x^2 + 1}} - \frac{1}{6}$$

$$t'(x) = 0 \Leftrightarrow \frac{x}{3\sqrt{x^2 + 1}} = \frac{1}{6}$$

$$\Rightarrow 3x^2 - 1 = 0$$

$$\Rightarrow x = \pm \frac{\sqrt{3}}{3}$$

$$\Rightarrow x = \frac{\sqrt{3}}{3} \text{ km} \\ \downarrow \\ 0,577 \text{ km} \quad (4)$$

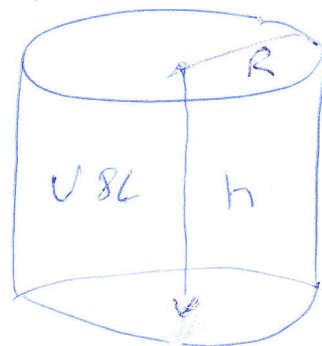


Exo 3

$$V = \pi R^2 h$$

$$1\ell = 1000 \text{ cm}^3$$

$$8\ell = 8000 \text{ cm}^3$$



$$S = S_{\text{cylindre}} + S_{\text{base}}$$

$$S(R, h) = 2\pi R \cdot h + \pi R^2$$

$$\text{on a : } \pi R^2 h = 8000$$

$$\Rightarrow h = \frac{8000}{\pi R^2}$$

$$\text{donc } S_{(h)} = 2\pi R \cdot \frac{8000}{\pi R^2} + \pi R^2$$

$$S(R) = \frac{16000}{R} + \pi R^2 \quad (1.5)$$

$$S' = \left(\frac{16000}{R} \right)' + (\pi R^2)'$$

$$S'(R) = -\frac{16000}{R^2} + 2\pi R \quad (1.5)$$

$$S' = 0 \Rightarrow 2\pi R = \frac{16000}{R^2}$$

$$\Rightarrow 2\pi R^3 = 16000$$

$$\Rightarrow R = \sqrt[3]{\frac{16000}{2\pi}}$$

$$R = 13,6557 \text{ cm}$$

$$h = \frac{8000}{\pi R^2} = 13,6557 \text{ cm} \quad (2)$$

$$\text{donc } h = R = 13,6557 \text{ cm}$$